COLLISION INTEGRALS OF IONIZED AIR COMPONENTS FOR A SCREENED COULOMB POTENTIAL

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Results of a numerical evaluation of the integrals $\Omega^{l,s}$ ($l = 1, 2, 3, 4$ and $s = l \ldots 8 - l$), in terms of which transport coefficients are expressed [1], are given in this paper. The results obtained are of practical use for calculating kinetic properties of ionized gases within the Chapman–Enskog theory, including third-order terms in the Sonine polynomial expansion. The integrals $\Omega^{l,s}$ were calculated for singly-ionized colliding air particles at temperatures T = 10,000-40,000°K and pressures p = 0.1, 1, and 100 atm. The screened Coulomb potential for attraction and repulsion was the model for electron–ion and ionic interactions, and the Debye screening length was chosen by taking into account screening both by electrons and by multiply-charged ions. Quantum effects are not important in the temperature and pressure ranges considered for air, and can, therefore, be neglected in calculating kinetic properties.

1. Transport processes in ionized gases can at present be calculated within the approximations of the Chapman–Enskog theory [1] if the screened coulomb potential is used for the charged particles interaction model. As shown by Grad [2] and Devoto [3], the solution for ionized gases converges if one retains at least three terms in the Sonine polynomial expansion. In this case the electric and thermal conductivities coincide with Spitzer's asymptotic expressions [4] for a fully ionized gas, obtained by numerical solution of the Fokker–Planck equation. The presence of three terms in the Sonine polynomial expansion requires the evaluation of transport cross sections Q^l of order $l = 1, 2, 3, 4$ and of integrals $\Omega^{l,s}$ of order s from $s = l$ to $s = 8 - l$. Quantum-mechanical corrections related to diffraction effects and particle indistinguishability were considered in great detail by Williams and Dewitt [5]. Quantum effects become important at sufficiently high temperatures, when the de Broglie wavelength becomes greater than the classical distance of closest approach. In the temperature range of interest for transport properties of air, quantum-mechanical corrections to the transport cross sections are unimportant.

A calculation of the integrals $\Omega^{l,s}$ for particles interacting through a screened Coulomb potential was performed in a number of papers [6-8]. However, the calculations by Devoto [6] and Liboff [7] are approximate. This enabled to obtain analytic expressions for the integrals $\Omega^{l,s}$

$$\Omega^{l,s} = A^{l,s} / T^2 (\ln \Lambda + B^{l,s}) \tag{1.1}$$

which can be used as asymptotic expressions for $\Lambda \gg 1$ ($\Lambda = H/E$, where H is the Debye screening length, and $E = e^2 z_1 z_2/kT$), where $A^{l,s}$ and $B^{l,s}$ are constants for given values of l and s. The results of the calculation, for a low-temperature, low-density gas and for a dense plasma at all temperatures, differ for attractive and repulsive potentials and deviate from the analytic expression (1.1). An exact numerical calculation of the integrals $\Omega^{l,s}_{\mp}$ was performed by Mason et al. [8] for a wide range of the parameter Λ. However, not all cross sections, required for the calculation of kinetic properties, are given there to third-order Sonine polynomials within the Chapman–Enskog theory.

2. The interaction of two colliding charged particles is described, for attraction and repulsion, by the screened Coulomb potential

$$\varphi(r) = \mp e^2 z_1 z_2 r^{-1} \exp(-r/H) \tag{2.1}$$

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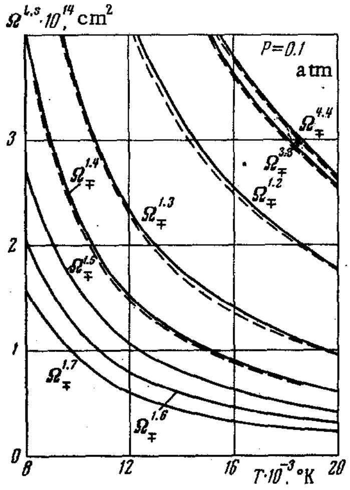

Fig. 1

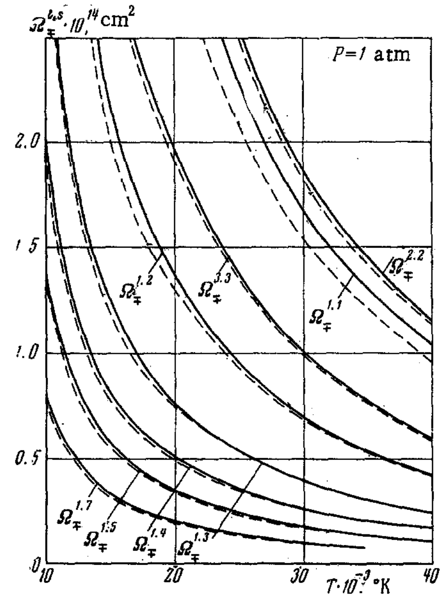

Fig. 2

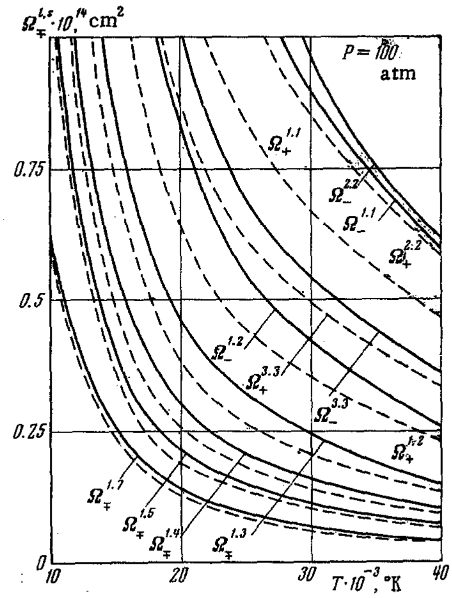

Fig. 3

where the plus sign corresponds to repulsion and the minus sign to attraction,

$$H = \sqrt{kT / 4\pi n_e e^2 \left(1 + \frac{1}{n_e} \Sigma n_i z_i^2\right)}$$

and H is the Debye screening length, taking into account the electrostatic effect of all charged particles (e is the electron charge, z_i is the ionic charge of type i, n_i is the number of type-i ions per unit volume, and n_e is the number of electrons per unit volume).

In the classical picture the trajectory of each particle is completely determined and the scattering is characterized by the angle of deflection χ

$$\chi = \pi - 2b \int_{r_m}^{\infty} \frac{dr}{r^2 \sqrt{1 - b^2/r^2 - 2\varphi(r)/\mu g^2}}$$

where b is the impact parameter, μ is the reduced mass of both colliding particles, g is the relative velocity of both particles at infinity, and r_m is the distance of closest approach, for which the radicand vanishes. The cross section Q^l is of the form

$$Q^l = 2\pi \int_0^{\infty} [1 - \cos^l(\chi)]\, b\, db \qquad (2.2)$$

The integrals $\Omega^{l,s}$, appearing in the transport coefficient expressions, are determined by a statistical average over the cross section Q^l

$$\Omega^{l,s} = \frac{4(l+1)}{(s+1)!\,[4l + 1 - (-1)^l]} \int_0^{\infty} \exp(-\gamma^2)\, \gamma^{2s+3} Q^l(g)\, d\gamma \qquad (2.3)$$

where $\gamma = \sqrt{\mu g^2/2kT}$ is the reduced relative velocity of the particles.

The method of numerical evaluation of the collision integrals is discussed by Smith and Munn [9]. The difficulties due to a long-range, screened potential are discussed by Mason et al. [8]. The numerical evaluation of the integrals (2.2) and (2.3) was performed here with an accuracy of 1%. The agreement of our results with the integrals $\Omega_{-}^{l,s}$ and $\Omega_{+}^{l,s}$ for attractive and repulsive potentials [8] is within the numerical accuracy. The air composition data were taken from [10, 11].

The calculations of the integrals $\Omega_{\mp}^{l,s}$ for values of l and s, shown in Figs. 1-3, are for singly-charged air components in the temperature range T from 10,000 to 40,000°K and for pressures p = 0.1, 1, and 100 atm. The full curves correspond to attraction and the dashed curves to repulsion. For low pressures (p = 0.1 atm) the difference between the integrals $\Omega_{-}^{l,s}$ and $\Omega_{+}^{l,s}$, corresponding to attraction and repulsion potentials, is less than 5-6% for $l = 1$ and $l = 2$, and within 1-2% for higher l and s. The parameter Λ increases with decreasing air pressure, and for $p \le 0.01$ atm the value of Λ is at least larger than 100 for the whole temperature interval considered, so that the difference between attraction and repulsion in the potential (2.1) leads to an insignificant deviation of the integrals from their asymptotic values [6]. The value of Λ drops with increasing pressure: for a pressure of 10 atm and for $6000°K \le T \le 40{,}000°K$ the value of Λ varies from 6 to 40, and for a pressure of 100 atm Λ varies from 2.5 to 17 in the same temperature interval. It is seen from the values of $\Omega_{\mp}^{l,s}$ in Fig. 3 that the difference between $\Omega_{-}^{l,s}$ and $\Omega_{+}^{l,s}$ exceeds 1% for all values of l and s. For $\Omega_{\pm}^{11}$ and $\Omega_{\pm}^{22}$ this difference is about 30%.

Quantum corrections do not exceed 0.3% for the temperature and pressure ranges considered here.

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